Chapter 16: Multivariate analysis of variance (MANOVA)

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Please, Sir, can I have some more … maths?

Calculation of E^{-1}

$$E = \begin{pmatrix} 51 & 13 \\ 13 & 122 \end{pmatrix}$$

determinant of $E, |E| = (51 \times 122) - (13 \times 13) = 6053$

matrix of minors for $E = \begin{pmatrix} 122 & 13 \\ 13 & 51 \end{pmatrix}$

pattern of signs for 2×2 matrix $= \begin{pmatrix} + & - \\ - & + \end{pmatrix}$

matrix of cofactors $= \begin{pmatrix} 122 & -13 \\ -13 & 51 \end{pmatrix}$

The inverse of a matrix is obtained by dividing the matrix of cofactors for E by $|E|$, the determinant of E:

$$E^{-1} = \begin{pmatrix} \dfrac{122}{6053} & \dfrac{-13}{6053} \\ \dfrac{-13}{6053} & \dfrac{51}{6053} \end{pmatrix} = \begin{pmatrix} 0.0202 & -0.0021 \\ -0.0021 & 0.0084 \end{pmatrix}$$

Calculation of HE^{-1}

$$HE^{-1} = \begin{pmatrix} 10.47 & -7.53 \\ -7.53 & 19.47 \end{pmatrix} \begin{pmatrix} 0.0202 & -0.0021 \\ -0.0021 & 0.0084 \end{pmatrix}$$
$$= \begin{pmatrix} [(10.47 \times 0.0202) + (-7.53 \times (-0.0021))] & [(10.47 \times (-0.0021)) + (-7.53 \times 0.0084)] \\ [(-7.53 \times 0.0202) + (19.47 \times (-0.0021))] & [(-7.53 \times (-0.0021)) + (19.47 \times 0.0084)] \end{pmatrix}$$
$$= \begin{pmatrix} 0.2273 & -0.0852 \\ -0.1903 & 0.1794 \end{pmatrix}$$

Calculation of eigenvalues

The eigenvalues or roots of any square matrix are the solutions to the determinantal equation $|A - \lambda I| = 0$, in which A is the square matrix in question and I is an identity matrix of the same size as A. The number of eigenvalues will equal the number of rows (or columns) of the square matrix. In this case the square matrix of interest is HE^{-1}:

$$
\begin{aligned}
|HE^{-1} - \lambda I| &= \left| \begin{pmatrix} 0.2273 & -0.0852 \\ -0.1930 & 0.1794 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| \\
&= \left| \begin{pmatrix} (0.2273 - \lambda) & -0.0852 \\ -0.1930 & (0.1794 - \lambda) \end{pmatrix} \right| \\
&= [(0.2273 - \lambda)(0.1794 - \lambda) - (-0.1930 \times (-0.0852))] \\
&= \lambda^2 - 0.2273\lambda - 0.1794\lambda + 0.0407 - 0.0164 \\
&= \lambda^2 - 0.4067\lambda + 0.0243
\end{aligned}
$$

Therefore the equation $|HE^{-1} - \lambda I| = 0$ can be expressed as:

$$
\lambda^2 - 0.4067\lambda + 0.0243 = 0
$$

To solve the roots of any quadratic equation of the general form $a\lambda^2 + b\lambda + c = 0$ we can apply the following formula:

$$
\lambda_i = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}
$$

For the quadratic equation obtained, $a = 1$, $b = -0.4067$, $c = 0.0243$. If we replace these values into the formula for discovering roots, we get:

$$
\begin{aligned}
\lambda_i &= \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \\
&= \frac{0.4067 \pm \sqrt{[(-0.4067)^2 - 0.0972]}}{2} \\
&= \frac{0.4067 \pm 0.2612}{2} \\
&= \frac{0.6679}{2} \text{ or } \frac{0.1455}{2} \\
&= 0.334 \text{ or } 0.073
\end{aligned}
$$

Hence, the eigenvalues are 0.334 and 0.073.